

Particle production in models with helicity-0 graviton ghost in de Sitter spacetime

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We revisit the problem of the helicity-0 ghost mode of massive graviton in the de Sitter background. In general, the presence of a ghost particle, which has negative energy, drives the vacuum to be unstable through pair production of ghost particles and ordinary particles. In the case that the vacuum state preserves the de Sitter invariance, the number density created by the pair production inevitably diverges due to unsuppressed ultra-violet(UV) contributions. In such cases one can immediately conclude that the model is not viable. However, in the massive gravity theory we cannot construct a vacuum state which respects the de Sitter invariance. Therefore the presence of a ghost does not immediately mean the breakdown of the model. Explicitly estimating the number density and the energy density of particles created by the pair production of two conformal scalar particles and one helicity-0 ghost graviton, we find that these densities both diverge. However, since models with helicity-0 ghost graviton have no de Sitter invariant vacuum state, it is rather natural to consider a UV cutoff scale in the three-dimensional momentum space. Then, even if we take the cutoff scale as large as the Planck scale, the created number density and energy density are well suppressed. In many models the cutoff scale is smaller than the Planck scale. In such models the created number density and the energy density are negligibly small as long as only the physics below the cutoff scale is concerned.

I. INTRODUCTION

The present accelerated expansion of the Universe is one of the hottest topics in cosmology [1]. In order to explain it, various modified models of cosmology have been proposed and studied. We roughly classify them into two categories. One consists of models which utilize the spin-0 sector [3]. Representative examples in this category are the cosmological constant [2] and the quintessential models [4]. The other category consists of models which use the spin-2 sector. Most of modified gravity theories in this category fall into a massive gravity theory with higher order coupling terms. Then, in most cases the mass of the graviton should be tuned to the same order as the present Hubble parameter H_0 if we try to explain the accelerated expansion of the universe. However, it is known that the helicity-0 mode of the graviton becomes a ghost mode in the de Sitter background with the Hubble parameter H when the graviton mass is in the range $0 < m^2 < 2H^2$ [5]. It is often said that the existence of a ghost mode immediately implies that the model is not viable.

The disaster caused by a ghost mode is easily understood in the Minkowski background. The excitation energy of a ghost mode is negative [6, 7]. If the ghost couples with an ordinary matter field whose excitation energy is positive, spontaneous pair production should occur since it is not forbidden by the energy conservation and the momentum conservation. One may think that, if the coupling is extremely suppressed, the pair production rate is negligibly small. However, this naive expectation is not true. If the initial vacuum state keeps the Lorentz symmetry unbroken, the probability of pair production is the same for the processes boosted by Lorentz transformation. In order to calculate the total creation rate, we must sum up the contributions from various processes labelled by the 3-dimensional momentum \mathbf{p} . Since the integrand is $\propto 1/p$, the integral is divergent due to UV contributions. It will be natural to expect that in the de Sitter background the same pathology will remain to exist since the UV behavior will not be affected by the presence of the spacetime curvature.

There are many attempts to construct a ghost free model in which a spin-2 field drives the accelerated expansion of the Universe. In this context DGP braneworld model [8] is the model which has been recently most extensively studied because it has a self-acceleration branch of cosmological solutions [9]. However, the self-acceleration branch of this model is thought to be unrealistic, since it has a ghost mode in view of a four-dimensional effective theory [10].

Nevertheless, we do not think that this is the end of the story. In our previous work we pointed out that there is

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no vacuum state which maintains the de Sitter invariance in general massive gravity theories with the helicity-0 ghost mode [7]. Hence, the above argument which leads to divergent pair production in the Minkowski spacetime does not apply to the helicity-0 ghost mode in the de Sitter background as it is. In this paper, we examine the number density and the energy density created by the spontaneous production of two conformal scalar particles together with one helicity-0 graviton. The results turn out to be divergent due to the UV contributions. However, in massive gravity theories often there exists a strong coupling scale beyond which the perturbative expansion is no more valid. The strong coupling scale has been studied in Ref. [11] for generic massive gravity theories and in Ref. [12] for DGP braneworld model with the Minkowski brane background. Since in the massive gravity theory the de Sitter invariance is already broken by choosing a vacuum state, the region of strong coupling in momentum space will be naturally specified not by the four-dimensional momentum but by the three-dimensional momentum. In order to exclude the contribution from the region of strong coupling, a cutoff scale for the three-dimensional momentum naturally arises. Even if we set the cutoff momentum to the Planck scale, the created energy density is not very large. The strong coupling scales estimated in literature are much smaller than the Planck value. As a result, such a model does not show violent particle production as long as we are restricted to the region where the perturbative expansion is valid.

This paper is organized as follows. In section II, we will introduce a model of massive gravity theory with a conformal scalar field. In section III, we discuss a transformation which simplifies the coupling term between the helicity-0 ghost mode and the scalar field. In section IV, we will give an estimate for the total number density and energy density of the created scalar particles. In section V, we will summarize the results.

II. SET UP

We consider a massive gravity theory whose action is given by [13]

$$S = m_{pl}^2 \int d^4x \sqrt{-g} \left(R - \frac{m^2}{4} (h^{\mu\nu} h_{\mu\nu} - h^2) + L_m \right), \quad (2.1)$$

where R is Ricci scalar, $h_{\mu\nu} \equiv g_{\mu\nu} - g_{\mu\nu}^{(0)}$ and L_m is the matter Lagrangian. This mass term gives the only spin-2 propagation of the graviton [13]. The background spacetime is given by the de Sitter metric;

$$\begin{aligned} ds^2 &= \left(\frac{-1}{H\eta} \right)^2 (-d\eta^2 + dx^2 + dy^2 + dz^2) \\ &\equiv g_{\mu\nu}^{(0)} dx^\mu dx^\nu. \end{aligned} \quad (2.2)$$

Since we are interested in the case with a ghost mode, we assume that the mass of the graviton is in the range $0 < m^2 < 2H^2$, where the helicity-0 mode of the graviton becomes a ghost mode [5]. When we take the flat slicing of the de Sitter background, the helicity-0 mode of the graviton can be written as [5]

$$h_{00}(\mathbf{x}, \eta) = \int d^3p \frac{2p^2}{m_{pl} m \sqrt{3(2H^2 - m^2)}} \left(a^\dagger(\mathbf{k}) f_{\mathbf{p}}^{m^2}(\eta) e^{i\mathbf{p} \cdot \mathbf{x}} + \text{h.c.} \right), \quad (2.3)$$

$$h_{0i}(\mathbf{x}, \eta) = \frac{\partial_i}{\Delta} \left[h'_{00}(\mathbf{x}) - \frac{2}{\eta} h_{00}(\mathbf{x}) \right], \quad (2.4)$$

$$\begin{aligned} h_{ij}(\mathbf{x}, \eta) &= \frac{\partial_i \partial_j}{\Delta^2} \left[\Delta h_{00}(\mathbf{x}) - \frac{3}{2} \left(\frac{2}{\eta} h'_{00}(\mathbf{x}) + \frac{m^2 - 6H^2}{(H\eta)^2} h_{00}(\mathbf{x}) \right) \right] \\ &\quad + \frac{1}{2} \frac{1}{\Delta} \eta_{ij} \left[\frac{2}{\eta} h'_{00}(\mathbf{x}) + \frac{m^2 - 6H^2}{(H\eta)^2} h_{00}(\mathbf{x}) \right], \end{aligned} \quad (2.5)$$

with

$$f_{\mathbf{k}}^{m^2}(\mathbf{x}, \eta) = \left(\frac{\pi H^2}{4} e^{-\pi I m(\nu)} \right)^{1/2} \eta^{\frac{3}{2}} H_\nu^{(2)}(k\eta), \quad (2.6)$$

where a prime “ ’ ” denotes a differentiation with respect to η . The metric component h_{00} satisfies the equation of the Klein-Gordon type

$$\left(-\partial_\eta^2 + \partial_x^2 + \partial_y^2 + \partial_z^2 + \frac{2}{\eta} \partial_\eta - \frac{m^2}{(H\eta)^2} \right) h_{00} = 0. \quad (2.7)$$

Notice that, in order to keep the commutation relation $[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = \delta^3(\mathbf{k} - \mathbf{k}')$, it is necessary to associate the positive (negative) frequency mode functions in the normal sense with the creation (annihilation) operators $a^\dagger(\mathbf{k}')$ ($a(\mathbf{k})$) [7].

For simplicity, we consider a conformal scalar field as a normal non-ghost matter field. The action is given by

$$S_m = \int d^4x \frac{1}{2} \sqrt{-g} \left\{ g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{6} R \phi^2 \right\}, \quad (2.8)$$

which inherently has the coupling to the graviton. Following the canonical quantization, we can write ϕ [5]

$$\phi = \int d^3k \left(b(\mathbf{k}) \Phi(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}} + (\text{h.c.}) \right), \quad (2.9)$$

where

$$\Phi(\mathbf{k}) = f_{\mathbf{k}}^{2H^2}(\mathbf{x}, \eta) = \frac{H}{\sqrt{2k}} \eta \exp(-ik\eta), \quad (2.10)$$

and the creation and annihilation operators satisfy

$$[b(\mathbf{k}), b^\dagger(\mathbf{k}')] = \delta^3(\mathbf{k} - \mathbf{k}'), \quad (2.11)$$

$$[b(\mathbf{k}), b(\mathbf{k}')] = [b^\dagger(\mathbf{k}), b^\dagger(\mathbf{k}')] = 0. \quad (2.12)$$

The leading order coupling between the conformal scalar field and the graviton can be deduced from Eq.(2.8) as

$$S_{int} = -\frac{1}{2} \int d^4x \sqrt{-g^{(0)}} h^{\mu\nu} T_{\mu\nu}, \quad (2.13)$$

where

$$T_{\mu\nu} = \frac{2}{3} \phi_{;\mu} \phi_{;\nu} - \frac{1}{6} g_{\mu\nu} g^{\rho\sigma} \phi_{;\rho} \phi_{;\sigma} - \frac{1}{3} \phi_{;\mu\nu} \phi + \frac{1}{12} g_{\mu\nu} \phi \square \phi, \quad (2.14)$$

is the energy-momentum tensor of the conformal scalar field.

III. TRANSFORMATION OF COUPLING TERM

The mode function of the helicity-0 mode of the graviton with the three dimensional momentum $\mathbf{p} = (p, 0, 0)$ can be written as

$$h_{\mu\nu}(\mathbf{p}) = \begin{pmatrix} f_1 & ipf_2 & 0 & 0 \\ ipf_2 & -p^2 f_3 + f_4 & 0 & 0 \\ 0 & 0 & f_4 & 0 \\ 0 & 0 & 0 & f_4 \end{pmatrix} a^\dagger(\mathbf{k}) \exp(ipx) + (\text{h.c.}), \quad (3.1)$$

where

$$f_1(p, \eta) = \frac{2p^2}{m_{pl} m \sqrt{3(2H^2 - m^2)}} f_{\mathbf{p}}^{m^2}(\eta), \quad (3.2)$$

$$f_2(p, \eta) = -\frac{1}{p^2} \left[f_1'(p, \eta) - \frac{2}{\eta} f_1(p, \eta) \right], \quad (3.3)$$

$$f_3(p, \eta) = -\frac{1}{p^4} \left[p^2 f_1(p, \eta) + \frac{3}{2} \left(\frac{2}{\eta} f_1'(p, \eta) + \frac{m^2 - 6H^2}{(H\eta)^2} f_1(p, \eta) \right) \right], \quad (3.4)$$

$$f_4(p, \eta) = -\frac{1}{2p^2} \left[\frac{2}{\eta} f_1'(p, \eta) + \frac{m^2 - 6H^2}{(H\eta)^2} f_1(p, \eta) \right]. \quad (3.5)$$

We consider the transformation defined by

$$\hat{h}_{\mu\nu} = h_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu. \quad (3.6)$$

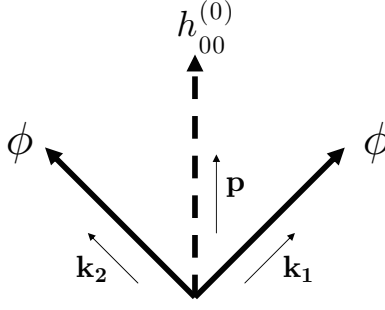


FIG. 1: a diagram of the lowest order pair creation of $h^{(0)}$ and ϕ .

Because the massive gravity theory has no gauge degree of freedom, this transformation changes the form of the action. However, using the conservation law $\nabla^\mu T_{\mu\nu} = 0$, one can show

$$\int d^4x \sqrt{-g} (\nabla^\mu \xi^\nu) T_{\mu\nu} = \int d^4x \sqrt{-g} \xi^\nu \nabla^\mu T_{\mu\nu} = 0.$$

This means that the shape of the leading order interaction terms does not change under this transformation. Using this relation, we simplify the lowest order coupling (4.5)¹. The helicity-0 component of ξ can be written as

$$\xi_\mu = (A, ipB, 0, 0) \exp(-ipx). \quad (3.7)$$

By setting

$$A = -f_2 - \frac{1}{2}f'_3 - \frac{1}{\eta}f_3 \quad \text{and} \quad B = -\frac{f_3}{2}, \quad (3.8)$$

we obtain

$$\hat{h}_{\mu\nu} = \text{diag}\left(f_1 + \frac{2}{\eta}A + 2A', f_4 + \frac{2}{\eta}A, f_4 + \frac{2}{\eta}A, f_4 + \frac{2}{\eta}A\right) + (\text{h.c.}). \quad (3.9)$$

We decompose $\hat{h}_{\mu\nu}$ into the pure trace component $h_{\mu\nu}^T$ and the $\{00\}$ -component $h_{\mu\nu}^S$ as

$$\hat{h}_{\mu\nu} = h_{\mu\nu}^T + h_{\mu\nu}^S, \quad (3.10)$$

$$h_{\mu\nu}^T = \text{diag}\left(-f_4 - \frac{2}{\eta}A, f_4 + \frac{2}{\eta}A, f_4 + \frac{2}{\eta}A, f_4 + \frac{2}{\eta}A\right) + (\text{h.c.}), \quad (3.11)$$

$$h_{\mu\nu}^S = \text{diag}\left(f_1 + f_4 + \frac{4}{\eta}A + 2A', 0, 0, 0\right) + (\text{h.c.}). \quad (3.12)$$

Then, as the energy-momentum tensor of the conformal scalar field is traceless, the interaction term becomes

$$S_{int} = -\frac{1}{2} \int d^4x h_{00}^S T_{00}. \quad (3.13)$$

and from Eqs. (2.3)-(2.6) we have

$$h_{00}^S = \int d^3p \frac{m\sqrt{3(2H^2 - m^2)}}{m_{pl}H^4\eta^4 p^2} \left(a^\dagger(\mathbf{k}) f_{\mathbf{k}}^{m^2}(x, \eta) + h.c.\right), \quad (3.14)$$

where we used the on-shell condition (2.7)².

¹ The change caused by this transformation is to add total derivative terms with respect to the time coordinate η . This change corresponds to the canonical transformation of variables.

² As we consider only the lowest order effect in coupling in this paper, Feynman diagrams containing internal loops are neglected. Therefore the interaction term can be rewritten by using the on-shell condition (2.7).

IV. PARTICLE CREATION FROM THE VACUUM

In this section, we estimate the number density of ϕ -particles created through the process lowest order in coupling, which is diagrammatically expressed in Fig. 1. The total number of the created ϕ -particles with the momentum \mathbf{k} will be evaluated by taking the expectation value of the number operator $N_{\phi,\mathbf{k}} \equiv b^\dagger(\mathbf{k})b(\mathbf{k})$, which is calculated at the leading order of perturbation as³

$${}_{in}\langle 0 | N_{\phi,\mathbf{k}} | 0 \rangle_{in} = \int d^3k_1 d^3k_2 d^3p_1 d^3k_3 d^3k_4 d^3p_2 \langle 0 | S_{int} | \mathbf{k}_1 \mathbf{k}_2 \mathbf{p}_1 \rangle \times \langle \mathbf{k}_1 \mathbf{k}_2 \mathbf{p}_1 | N_{\mathbf{k}} | \mathbf{k}_3 \mathbf{k}_4 \mathbf{p}_2 \rangle \langle \mathbf{k}_3 \mathbf{k}_4 \mathbf{p}_2 | S_{int} | 0 \rangle, \quad (4.1)$$

where $\{k_i\}$ and $\{p_i\}$ are the momenta of ϕ -particles and ghost particles, respectively. Using the relations

$$\Phi'^* = \frac{1}{\eta} \Phi^* + ik \Phi^*, \quad (4.2)$$

$$\Phi''^* = ik \frac{2}{\eta} \Phi^* - k^2 \Phi^*, \quad (4.3)$$

derived from Eq. (2.10), and the momentum conservation law

$$\mathbf{k}_1 \cdot \mathbf{k}_2 = \frac{p^2 - k_1^2 - k_2^2}{2}, \quad (4.4)$$

we can evaluate the matrix element as

$$\begin{aligned} & \langle \mathbf{k}_1, \mathbf{k}_2, \mathbf{p}_1 | S_{int} | 0 \rangle \\ &= \langle \mathbf{k}_1, \mathbf{k}_2, \mathbf{p}_1 | \int_{-\infty}^{\eta_f} d\eta d^3k_3 d^3k_4 d^3p_2 \lambda(\mathbf{k}_3, \mathbf{k}_4, \mathbf{p}_2) \delta^{(3)}(\mathbf{k}_3 + \mathbf{k}_4 - \mathbf{p}_2) b^\dagger(\mathbf{k}_3) b^\dagger(\mathbf{k}_4) a^\dagger(\mathbf{p}_2) | 0 \rangle \\ &= 2 \int_{-\infty}^{\eta_f} d\eta \lambda(\mathbf{k}_1, \mathbf{k}_2, \mathbf{p}_1) \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{p}_1), \end{aligned}$$

where

$$\lambda(\mathbf{k}_1, \mathbf{k}_2, \mathbf{p}) = -\frac{m\sqrt{2H^2 - m^2}(3(k_1 - k_2)^2 - p^2)}{16\sqrt{3}m_{pl}H^2\sqrt{k_1k_2}p^2\eta^2} f_{\mathbf{p}}^{m^2} e^{i(k_1+k_2)\eta}. \quad (4.5)$$

On the other hand, $\langle \mathbf{k}_1, \mathbf{k}_2, \mathbf{p}_1 | N_{\mathbf{k}} | \mathbf{k}_3, \mathbf{k}_4, \mathbf{p}_2 \rangle$ is given by

$$\begin{aligned} \langle \mathbf{k}_1, \mathbf{k}_2, \mathbf{p}_1 | N_{\mathbf{k}} | \mathbf{k}_3, \mathbf{k}_4, \mathbf{p}_2 \rangle &= \langle 0 | a(\mathbf{p}_1) b(\mathbf{k}_2) b(\mathbf{k}_1) b^\dagger(\mathbf{k}) b(\mathbf{k}) b^\dagger(\mathbf{k}_3) b^\dagger(\mathbf{k}_4) a^\dagger(\mathbf{p}_2) | 0 \rangle \\ &= \delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_2) \left\{ \delta^{(3)}(\mathbf{k}_1 - \mathbf{k}) + \delta^{(3)}(\mathbf{k}_2 - \mathbf{k}) \right\} \\ &\quad \times \left\{ \delta^{(3)}(\mathbf{k}_1 - \mathbf{k}_3) \delta^{(3)}(\mathbf{k}_2 - \mathbf{k}_4) + \delta^{(3)}(\mathbf{k}_1 - \mathbf{k}_4) \delta^{(3)}(\mathbf{k}_2 - \mathbf{k}_3) \right\}. \end{aligned} \quad (4.6)$$

Combining Eqs. (4.1), (4.5) and (4.6), we find that the number of ϕ -particles per unit comoving volume is given by

$$\begin{aligned} n_{com} &\equiv \int d^3k \frac{{}_{in}\langle 0 | N_{\mathbf{k}} | 0 \rangle_{in}}{\delta^{(3)}(0)} \\ &= 16 \int d^3k d^3k_1 d^3p \left| \int_{-\infty}^{\eta_f} d\eta \lambda(\mathbf{k}, \mathbf{k}_1, \mathbf{p}) \right|^2 \delta^{(3)}(\mathbf{k} + \mathbf{k}_1 - \mathbf{p}). \end{aligned} \quad (4.7)$$

Since the pathology caused by the existence of a ghost mode is that the number or energy density of the created particles suffers from UV divergence, we concentrate on the behavior in the UV limit. In this limit, we can use the approximation

$$f_{\mathbf{p}}^{m^2} \simeq \frac{H\eta}{\sqrt{2}p} e^{-ip\eta}. \quad (4.8)$$

³ Generally speaking, in the canonical quantization, the interaction action S_{int} is not identical to the spacetime integral of the non-linear term of the Lagrangian if the kinetic terms are not canonical. However, in the computation at the leading order of perturbation the difference does not arise [14].

Then, we have

$$\int_{-\infty}^{\eta_f} d\eta \lambda(\mathbf{k}, \mathbf{k}_1, \mathbf{p}) \simeq -\frac{m\sqrt{2H^2 - m^2}(3(k - k_1)^2 - p^2)}{16\sqrt{6}m_{pl}H\sqrt{kk_1p^5}} \int_{-\infty}^{\eta_f} d\eta \frac{1}{\eta} e^{i(k+k_1-p)\eta}. \quad (4.9)$$

Since the background is not stationary, the energy conservation law does not hold. Hence, Eq. (4.9) contains the contributions not only from the ghost instability but also from the violation of the energy conservation law. The latter contribution exists even if there is no ghost excitation [15] and is divergent. However, we think that the divergence of this type is responsible for the uncertainty in the definition of a particle when the interaction is turned on. In fact, if we smoothly turn off the interaction before η_f , UV divergence disappears in the non-ghost case. In contrast, the UV contribution due to the presence of a ghost mode does not disappear even if we smoothly turn off the interaction. Such a contribution comes from the momentum region $(k + k_1 - p)|\eta_f| \ll 1$. In this region, roughly speaking, the integral in Eq. (4.9) is $O(1)$. Then, we can easily estimate Eq. (4.9) as

$$\int_{-\infty}^{\eta_f} d\eta \lambda(\mathbf{k}, \mathbf{k}_1, \mathbf{p}) \Big|_{\text{ghost contribution}} \cong \frac{m\sqrt{2H^2 - m^2}(3(k - k_1)^2 - (k + k_1)^2)}{m_{pl}H\sqrt{kk_1(k + k_1)^5}}. \quad (4.10)$$

Substituting this estimate into Eq. (4.7), we have

$$n_{com} \cong \int dk dk_1 d\cos\theta k^2 k_1^2 \frac{m^2(2H^2 - m^2)(k^2 - 4kk_1 + k_1^2)^2}{m_{pl}^2 H^2 k k_1 (k + k_1)^5}, \quad (4.11)$$

The range of the $\cos\theta$ -integral in the above expression is approximately given by $(k + k_1 - |\mathbf{k} + \mathbf{k}_1|)|\eta_f| \leq 1$, which leads to $1 - \frac{k+k_1}{kk_1|\eta_f|} \leq \cos\theta \leq 1$. Then the number density is estimated as

$$n_{com} = O\left(\frac{m^2(2H^2 - m^2)\Lambda^2}{m_{pl}^2 H^2 |\eta_f|}\right), \quad (4.12)$$

where we have introduced a cutoff Λ in the three-dimensional comoving momentum integral. This result means that the number density is quadratically divergent. In the same manner the energy density of the created ϕ -particles is also evaluated as

$$\rho_{com} \cong \int d^3k \frac{{}_{in}\langle 0 | N_{\mathbf{k}} k | 0 \rangle_{in}}{\delta^{(3)}(0)} = O\left(\frac{m^2(2H^2 - m^2)\Lambda^3}{m_{pl}^2 H^2 |\eta_f|}\right). \quad (4.13)$$

V. DISCUSSION

We have studied the helicity-0 ghost of massive graviton in de Sitter space time. It is often said that the existence of a ghost mode immediately means that the model is not viable. In the Minkowski background this is because infinitely many particles are instantaneously created through the pair production of a ghost and a normal particle, irrespective of the strength of the interaction between the ghost and the normal particle. In de Sitter background the same phenomena is expected to occur because the UV behavior is almost the same. However, since the massive gravity theory with a ghost mode has no vacuum state which respects the de Sitter invariance, the argument used in the case of Minkowski background, which assumes the Lorentz invariance of the initial vacuum state, does not apply. Therefore, in this paper, we have explicitly evaluated the number density and the energy density of the particles created by the pair production, taking a conformal scalar field as the matter content which couples to the helicity-0 mode of the graviton.

The result was divergent due to the UV contribution. However, in the modified gravity theory, there is a natural cutoff momentum scale beyond which the model is strongly coupled. Since non-perturbative effects become important beyond the strong coupling scale, the region in the momentum space where the linear theory is justified is restricted. Since the vacuum state does not have de Sitter invariance, it is not so strange even if the model has a three-dimensional momentum cutoff instead of the usual four-dimensional covariant one. If the three-dimensional momentum cutoff is set to the Planck scale, which means $\Lambda H \eta_f = m_{pl}$, the proper energy density of the created particles becomes

$$\rho = \rho_{com} H^4 \eta_f^4 \lesssim O(H^3 m_{pl}), \quad (5.1)$$

where we used the fact that both m^2 and $(2H^2 - m^2)$ are $\lesssim O(H^2)$. This energy density is much smaller than the critical energy density of the universe $\rho_{crit} = H^2 m_{pl}^2$. Since we expect that the three-dimensional momentum cutoff scale is generally smaller than the Planck scale, the particle creation is extremely suppressed unless we extrapolate the result of perturbative analysis beyond its validity region. In order to discuss the possible hazardous nature of the helicity-0 ghost in the large momentum region, we need a method to handle non-perturbative quantum effects. We think without such a method one cannot conclude that models with a helicity-0 ghost in the massive gravity theory are all to be excluded.

In this paper, for simplicity, we have considered only a conformal scalar field. However, there could be a possibility that the UV behavior is worse for non-conformal fields because the conformal invariance eliminates the coupling through the trace of the energy momentum tensor. We will discuss the cases of more generic matter contents in future.

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